



5 Inner-product vector spaces

Douglas Wilhelm Harder, LEL, M.Math.

dwharder@uwaterloo.ca

dwharder@gmail.com





Introduction

- In this topic, we will introduce the next chapter
 - Defining the inner product for finite-dimensional vectors
 - Describing the norm *induced* by this inner product
 - Considering inner products on other vector spaces
 - Defining orthogonality of vectors
 - Considering orthogonality in other vector spaces
 - Revisiting the Pythagorean theorem
 - Describing projections and best approximations
 - Describing the Cauchy-Bunyakovsky-Schwarz inequality
 - Defining the angle between vectors
 - Describing the Gram-Schmidt algorithm





Inner products

- An *inner product* $\langle \mathbf{u}, \mathbf{v} \rangle$ will be a measure of the *angle* between two vectors
 - We will use this to describe the relationship between them
 - We will find the best approximation of one vector by a scalar multiple of another
 - The angle will be found to be $\frac{\langle \mathbf{u}, \mathbf{v} \rangle}{\|\mathbf{u}\|_2 \|\mathbf{v}\|_2} = \cos(\theta)$
- In secondary school, you may have called an operation the *dot product*
 - Like we use the “2-norm” instead of the “Euclidean length”
 - We will use the *inner product* and not the *dot product*





Inner products

- We will use the angle-bracket notation $\langle \mathbf{u}, \mathbf{v} \rangle$
 - This is the notation you will use with wave functions in your course on quantum mechanics
- Many of you are already familiar with the dot notation $\mathbf{u} \cdot \mathbf{v}$
 - You will see this notation in calculus and physics
 - This is not a multiplication of two vectors
 - The result is a scalar value





Outcomes

- Following this chapter, you will be able to:
 - Calculate the inner product between two vectors
 - Describe the relationship between the inner product and the 2-norm
 - Use orthogonality and be able to apply the Pythagorean theorem
 - Find the best approximation of one vector by another using projections
 - Describe the angle between vectors
 - Apply the Gram-Schmidt algorithm





Summary

- Following this topic, you now
 - Are aware of what the next few sections will cover
 - Understand we will be measuring angle between vectors
 - Understand that we are using the term *inner product* and we are using the angled-bracket notation





References

- [1] https://en.wikipedia.org/wiki/Inner_product_space





Acknowledgments

None so far.





Colophon

These slides were prepared using the Cambria typeface. Mathematical equations use Times New Roman, and source code is presented using Consolas.

The photographs of flowers and a monarch butter appearing on the title slide and accenting the top of each other slide were taken at the Royal Botanical Gardens in October of 2017 by Douglas Wilhelm Harder. Please see

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for more information.





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